22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

MATHEMATICAL MODELING AND NONLINEAR CONTROL OF STRUCTURES UNDER ROTATION

Eduardo Palhares Júnior
Universidade Federal do ABC – Av dos Estados, 5001 – Bangu – Santo André
Eduardo.palharesjr@gmail.com
André Fenili
Universidade Federal do ABC – Av dos Estados, 5001 – Bangu – Santo André
Andre.fenili@ufabc.edu.br

Abstract. *This work investigates the dynamics and control of a rigid rod subjected to high speed rotations around a fixed axis and undergone gravity. The Mathematic model present intends to be a first approach for a helicopter system model, considering a static flight. Using the Lagrangian approach the nonlinear equations of motion are obtained. A spring and a damper coupled with a rigid rod are considered, with the goal of bringing the effects of flexibility and structural damping vibration, associated with a real helicopter rotor. A control technique based on feedback nonlinear terms (Feedback Linearization) is proposed in order to minimize the effects of vibration of the helicopter blades and guarantee a constant rotor speed, aiding the stability of the aircraft*

Keywords: *Non-linear systems, Helicopter, Non-linear control, Feedback linearization*

1. INTRODUCTION

Feedback linearization is a control technique used for nonlinear systems. It is viewed as a generalization of pole placement for linear systems [Marino and Tomei, 1995; Sheen and Bishop, 1992]. The basic idea of this approach to nonlinear control design is to algebraically transform a more complex nonlinear system dynamics into a simpler and equivalent linear one (completely or partly), so that well known linear control techniques can be applied [Isidori, 1995].

The existence of an output function h(x) used for feedback is essential to solving the feedback linearization problem. The necessary and sufficient conditions for the existence of h(x) involves the rank of a controllability matrix whose columns are composed by Lie brackets of vector fields associated to the system to be controlled and the concept of involutivity of a distribution which is formed by these same Lie brackets [Sheen and Bishop, 1992; Marquez, 2003] as discussed in this paper.

Feedback linearization is an approach to nonlinear control design that has attracted many researches in different fields [Singh and Yin, 1996; Joo and Seo, 1997; Sheen and Bishop, 1992, for example].

2. GEOMETRIC AND MATHEMATICAL MODEL

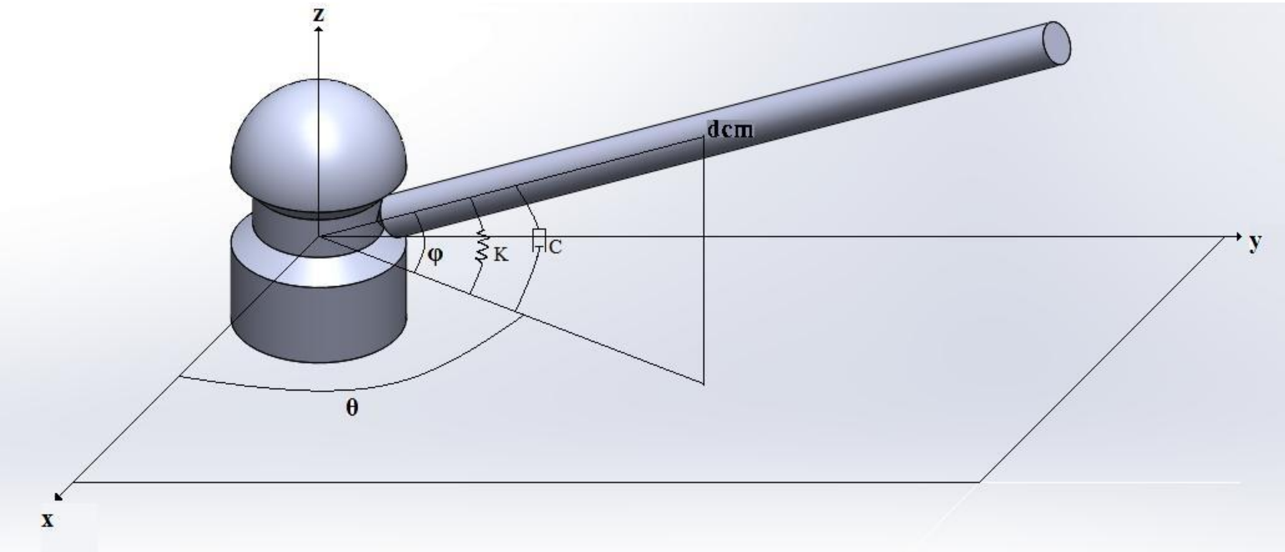

Figure 1. Geometric representation

The geometric model of the system investigated in this paper is presented in Figure 1. This system comprises a rigid rod rotating around a fixed axis and underwent gravity. Before proceeding, let us fix some notations.

m: mass of rod

d_{cm}: distance of mass center from the origin

I: inertia of rod

g: gravity acceleration

K: spring constant

C: damper constant

Considering that the system is in rotation around a fixed point, it's convenient does make a variable transformation for exploring the spherical symmetry of the system. Then, the transformation in (1) shows the convenient geometry of the problem.

$$r_{cm}(x,y,z) \to r_{cm}(d_{cm},\theta,\varphi) \quad (1)$$

$$x\hat{\imath} + y\hat{\jmath} + z\hat{k} \mapsto d_{cm}\cos\theta\cos\varphi\,\hat{\imath} + d_{cm}\sin\theta\cos\varphi\,\hat{\jmath} + d_{cm}\sin\varphi\,\hat{k}$$

Let the origin of the system with the rod in the horizontal position (azimuthal plane) and considering the gravity effect over the body. In this case, the system would make a harmonic motion because the dissipative effects are being neglected. Because of this, we added a torsional damper and spring to represent structural effects. The equations in (2) describe the kinetic and potential energy (respect) in the system.

$$T = T_{tr} + T_{rot} = \frac{m|\dot{r_{cm}}|^2}{2} + \frac{I|\dot{\theta}|^2}{2} + \frac{I|\dot{\varphi}|^2}{2} \quad (2)$$

$$V = mg|-r_z| + K\varphi^2 = mgd_{cm}\sin\varphi + K\varphi^2$$

Using the Lagrangian approach, it's possible to make an energy balance in the system to find the corresponding governing equations of the system. We will add the Rayleigh dissipation term that is related to the damper.

$$R = \frac{1}{2}C|\dot{\varphi}|^2 \quad (3)$$

Then, the Euler-Lagrange equation (4) gives us a explicitly method to make the balance.

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) + \frac{\partial F}{\partial \dot{q_i}} = 0 \quad (4)$$

Using the equations (4) in (2) and (3) we get the following system of equations.

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = md_{cm}{}^2(\ddot{\theta}\sin^2\varphi + 2\dot{\theta}\cos\varphi\sin\varphi\,\dot{\varphi}) + I\ddot{\theta} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} + \frac{\partial R}{\partial \dot{\varphi}} = md_{cm}{}^2\ddot{\varphi} + I\ddot{\varphi} - md_{cm}{}^2\dot{\theta}^2\sin\varphi\cos\varphi + mgd_{cm}\cos\varphi - K\varphi + 2C\dot{\varphi} = 0 \end{cases} \quad (5)$$

Adding the control law U acting in the equatorial angle and writing the equations in the linear form, we get these equations:

$$\begin{cases} \ddot{\theta} - \frac{2c_1\dot{\theta}\cos\varphi\sin\varphi\,\dot{\varphi}}{(c_1\cos^2\varphi + I)} = \frac{U}{(c_1\cos^2\varphi + I)} \\ \ddot{\varphi} + \frac{-c_1\dot{\theta}^2\sin\varphi\cos\varphi + c_3\cos\varphi - \frac{1}{2}K + 2C\dot{\varphi}}{c_2} = 0 \end{cases} \quad (6)$$

where:
$c_1 = md_{cm}{}^2$
$c_2 = md_{cm}{}^2 + I$
$c_3 = mgd_{cm}$

Using the coordinated transformation following (7) and writing Equations (6) in state space form (8):

$$\begin{cases} x_1 = \theta \\ x_2 = \dot{\theta} \\ x_3 = \varphi \\ x_4 = \dot{\varphi} \end{cases} \tag{7}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{2c_1 x_2 x_4 \cos\varphi \sin x_3 + U}{c_1 \cos^2 x_3 + I} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{c_1 {x_2}^2 \sin x_3 \cos x_3 - c_3 \cos x_3 + \frac{1}{2}K - 2Cx_4}{c_2} \end{cases} \tag{8}$$

3. THE VECTOR FIELDS F AND G

In order to check whether the proposed nonlinear control technique named feedback linearization can be applied to the nonlinear or not, the set of governing equations of motion in state space form as given by Equations (8) must be written in the form [Slotine and Li, 1991; Marino and Tomei, 1995]:

$$\dot{\boldsymbol{x}} = f(x) + ug(x) \tag{9}$$

Writing Equation (2) in this form results:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{Bmatrix} x_2 \\ \frac{2c_1 x_2 x_4 \cos\varphi \sin x_3}{c_1 \cos^2 x_3 + I} \\ x_4 \\ \frac{c_1 {x_2}^2 \sin x_3 \cos x_3 + c_3 \cos x_3 + \frac{1}{2}K - 2Cx_4}{c_2} \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{1}{c_1 \cos^2 x_3 + I} \\ 0 \\ 0 \end{Bmatrix} U \tag{10}$$

and the vector fields **f** and **g** are given by:

$$f(x) = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} \tag{11}$$

where:

$f_1 = x_2$

$f_2 = \dfrac{2c_1 x_2 x_4 \cos x_3 \sin x_3}{c_1 \cos^2 x_3 + I}$

$f_3 = x_4$

$f_4 = \dfrac{c_1 {x_2}^2 \sin x_3 \cos x_3 + c_3 \cos x_3 + \frac{1}{2}K - 2Cx_4}{c_2}$

and

$$g(x) = \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \end{Bmatrix} \tag{12}$$

where:

$C_{11} = C_{31} = C_{41} = 0$

$C_{21} = \dfrac{1}{c_1 \cos^2 x_3 + I}$

4. DEFINING THE LIE BRACKETS

The next step is to build the vector fields **g**, $ad_{\mathbf{f}}$ **g**, ... , $ad^{n-1}_{\mathbf{f}}$ **g** for the system of Equations (4). The notation $ad_{\mathbf{f}}$ **g** represents the Lie bracket of the vector fields f and g and defines a third vector as given by [Slotine and Li, 1991; Marino and Tomei, 1995]:

$$ad_f g(x) = [f, g] = \nabla g . f - \nabla f . g \tag{13}$$

For the case investigated here one has four states. Using the definition given by Equation (13) and the vectors f and g given by Equations (11) and (12) one has:

$$ad_f g = \begin{Bmatrix} C_{12} \\ C_{22} \\ C_{32} \\ C_{42} \end{Bmatrix} \tag{14}$$

where:

$$C_{12} = -\frac{1}{c_1 \cos^2 x_3 + I}$$
$$C_{22} = C_{32} = 0$$
$$C_{42} = -\frac{c_1 x_2 \sin 2x_3}{c_2 (c_1 \cos^2 x_3 + I)}$$

In the same way, one can show that:

$$ad_f{}^2 g = [f, ad_f g] = \nabla ad_f g . f - \nabla \mathrm{f} . ad_f g \tag{15}$$

or:

$$ad^2{}_f g = \begin{Bmatrix} C_{13} \\ C_{23} \\ C_{33} \\ C_{43} \end{Bmatrix} \tag{16}$$

where:

$$c_4 = (2\mathrm{I}^2 + 3c_1 \mathrm{I} + c_1{}^2)$$
$$C_{13} = -\frac{2c_1 x_4 \cos x_3 \sin x_3}{(c_1 \cos^2 x_3 + I)^2}$$
$$C_{23} = \frac{c_1{}^2 x_2{}^2 \sin^2 2x_3}{c_2 (c_1 \cos^2 x_3 + I)^2}$$
$$C_{33} = \frac{c_1 x_2 \sin 2x_3}{c_2 (c_1 \cos^2 x_3 + I)}$$
$$C_{43} = -\frac{c_1 x_2 (c_1 \mathrm{I} x_4 + c_1{}^2 x_4 + c_4 x_4 \cos 2x_3 + 4Cc_1 \cos^3 x_3 \sin x_3 + 2C\mathrm{I} \sin 2x_3 + c_1 \mathrm{I} x_4 \sin^2 2x_3 + c_1{}^2 x_4 \sin^2 2x_3)}{c_2{}^2 (\mathrm{I} + c_1 \cos^2 x_3)^2}$$

and, finally:

$$ad^3{}_f g = [f, ad_f{}^2 g] = \nabla ad_f{}^2 g . f - \nabla \mathrm{f} . ad_f{}^2 g \tag{17}$$

or:

$$ad_f{}^3 g = \begin{Bmatrix} C_{14} \\ C_{24} \\ C_{34} \\ C_{44} \end{Bmatrix} \tag{18}$$

where:

$$c_5 = I + c_1$$
$$c_6 = 2I + c_1$$
$$C_{14} = \frac{c_1 c_2 \left(\frac{x_2{}^2 (-2c_6 \cos 2x_3 + c_1 (\cos 4x_3 - 3))}{2(c_1 \cos^2 x_3 + I)} - (c_1 x_2{}^2 \sin^2 2x_3 - \sin 2x_3 (K - 4Cx_4 - 2c_3 \cos x_3 + c_1 \mathrm{x2}^2 \sin 2x_3))\right)}{c_2 (c_1 \cos^2 x_3 + I)^2}$$

$$C_{24} = \frac{c_1{}^2 x_2{}^2 (8Cc_1 \cos^4 x_3 \sin^2 x_3 + \sin 2x_3 (3c_1 c_5 x_4 + 2c_4 x_4 \cos 2x_3 - c_1 c_2 \mathrm{x4} \cos 4x_3 + 2C\mathrm{I} \sin 2x_3))}{c_2{}^2 (c_1 \cos^2 x_3 + I)^3}$$
$$C_{34} = \frac{2c_1 x_2 (c_1 c_5 x_4 + c_4 x_4 \cos 2x_3 + 2Cc_1 \cos^3 x_3 \sin x_3 + C\mathrm{I} \sin 2x_3 + c_1 c_5 x_4 \sin^2 2x_3)}{c_2{}^2 (c_1 \cos^2 x_3 + I)^2}$$
$$C_{44} = \frac{c_1 x_2}{2c_2{}^3 (c_1 \cos^2 x_3 + I)^3} (C_{44}^1 + C_{44}^2 + C_{44}^3 + C_{44}^4)$$

$$C_{44}^1 = -2c_2 (\mathrm{I} + c_1 \cos^2 x_3)^2 (c_1 x_2{}^2 \cos 2x_3 + c_3 \sin x_3) \sin 2x_3 - 2c_1{}^2 c_2 x_2{}^2 (\mathrm{I} + c_1 \cos^2 x_3) \sin^3 2x_3$$
$$C_{44}^2 = 8Cc_1 c_2 x_4 \sin 2x_3 (\mathrm{I} + c_1 \cos^2 x_3)(c_5 c_1 x_4 + c_4 x_4 \cos 2x_3 + 4Cc_1 \cos^3 x_3 \sin x_3 + 2C\mathrm{I} \sin 2x_3 + c_5 c_1 x_4 \sin^2 2x_3)$$
$$C_{44}^3 = -c_5 (\mathrm{I} + c_1 \cos^2 x_3)(K - 4Cx_4 - 2c_3 \cos x_3 + c_1 x_2{}^2 \sin 2x_3)(c_6 \cos 2x_3 + c_1 (1 + \sin^2 2x_3))$$
$$C_{44}^4 = -c_5 x_4 (6C\mathrm{I}c_1 + 3Cc_1{}^2 + 4Cc_4 \cos 2x_3 + Cc_1 (2\mathrm{I} + c_1) \cos 4x_3 - (8\mathrm{I}^3 + 16\mathrm{I}^2 c_1 + 2\mathrm{I}c_1{}^2 - 6c_1{}^3) x_4 \sin 2x_3 + (6\mathrm{I}^2 c_1 + 9\mathrm{I}c_1{}^2 + 3c_1{}^3) x_4 \sin 4x_3$$

5. THE CONTROLABILITY MATRIX

The controllability matrix for nonlinear systems is given by [Slotine and Li, 1991; Marino and Tomei, 1995]:

$$C = [g \quad ad_f g \quad {ad_f}^2 g \quad {ad_f}^3 g] \tag{19}$$

All the columns of the controllability matrix C are the Lie brackets given in Section 4. Substituting each one of these vectors in matrix C one obtains:

$$\mathrm{C}= \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \tag{20}$$

Since for the system analyzed in this work the elements of matrix C given by C_{21}, C_{31}, C_{41}, C_{51}, C_{22} and C_{42} are equal to zero, the matrix given in (12) can be rewritten as:

$$\mathrm{C}= \begin{bmatrix} 0 & C_{12} & C_{13} & C_{14} \\ C_{21} & 0 & C_{23} & C_{24} \\ 0 & 0 & C_{33} & C_{34} \\ 0 & C_{42} & C_{43} & C_{44} \end{bmatrix} \tag{21}$$

Calculating the determinant of C:

$$C = \frac{2\mathrm{c}_1{}^2 x_2{}^2}{\mathrm{c}_2{}^3(\mathrm{c}_6+\mathrm{c}_1\cos 2x_3)^5} * (-5\mathrm{c}_1{}^2 x_2{}^2 + (96\mathrm{I}^2 + 144\mathrm{I}\mathrm{c}_1 + 48\mathrm{c}_1{}^2) x_4{}^2 - 4\mathrm{c}_1\big(\mathrm{c}_1(x_2{}^2 - 16x_4{}^2) + 2\mathrm{I}(x_2{}^2 - 8x_4{}^2)\big)\cos 2x_3 + 4\big(8\mathrm{I}^2 x_4{}^2 + 12\mathrm{I}\mathrm{c}_1 x_4{}^2 + \mathrm{c}_1{}^2(x_2 + 4x_4{}^2)\big)\cos 4x_3 + (8\mathrm{I}\mathrm{c}_1 x_2{}^2 + 4\mathrm{c}_1{}^2 x_2{}^2)\cos 6x_3 + \mathrm{c}_1{}^2 x_2{}^2 \cos 8x_3 + 8I\mathrm{c}_3 \sin 2x_3 - 5\mathrm{c}_1\mathrm{c}_3 \sin x_3 - 8K\mathrm{c}_1 \sin 2x_3 + 64C\mathrm{c}_1 x_4 \sin 2x_3 - 4\mathrm{c}_3\mathrm{I}\sin 3x_3 - 9\mathrm{c}_1\mathrm{c}_3\sin 3x_3 + (64C\mathrm{I}x_4 + 32C\mathrm{c}_1 x_4 - 8\mathrm{I}K - 4\mathrm{K}\mathrm{c}_1)\sin 4x_3 - (12I\mathrm{c}_3 + 5\mathrm{c}_1\mathrm{c}_3)\sin 5x_3 - \mathrm{c}_1\mathrm{c}_3 \sin 7x_3) \tag{22}$$

We found out that this determinant is different from 0 (zero), so the matrix has rank 4 and then is complete in the region studied.

6. DETERMINATION OF THE LIE BRACKETS

Another condition to be satisfied in order to the rigid rod system to be input-state linearizable is that the distribution:

$$\Delta= span\{g, ad_f g, \ldots, ad_f^{n-2} g\} \tag{23}$$

be involutive near some equilibrium state [Slotine e Li, 1991]. This condition is a result of the Frobenius Theorem and guarantees the existence of a diffeomorphic transformation [Isidori, 1995]. The existence of a diffeomorphic transformation implies the existence of a 1-to-1 mapping from a nonlinear vector field to a linear vector field and vice-versa. In other words, if a Lie bracket is formed by two vectors (from a determined set, as the distribution presented in (23), for example) the vector field resulting from this operation can be expressed as a linear combination of the original set of vector fields [Isidori, 1995].

In this work, since there are four states, one must verify the involutivity of:

$$\Delta= span\{g, ad_f g, ad_f^2 g\} \tag{24}$$

In order to check the involutivity of the distribution given in (24), the following two steps must be followed [Isidori, 1995]:

Step A. The following Lie brackets must be determinated:
A.1. $[\mathbf{g}, \mathrm{ad}_{\mathbf{f}}\,\mathbf{g}]$
A.2. $[\mathbf{g}, \mathrm{ad}^2{}_{\mathbf{f}}\,\mathbf{g}]$
A.3. $[\mathbf{g}, \mathrm{ad}^3{}_{\mathbf{f}}\,\mathbf{g}]$

Step B. The existence of a_i and b_i must be proved such that:
B.1. $a_1\mathbf{g} + b_1\,\mathrm{ad}_{\mathbf{f}}\,\mathbf{g} = [\mathbf{g}, \mathrm{ad}_{\mathbf{f}}\,\mathbf{g}]$

B.2. $a_2 \mathbf{g} + b_2 \, ad^{2}{}_{\mathbf{f}} \mathbf{g} = [\mathbf{g}, ad^{2}{}_{\mathbf{f}} \mathbf{g}]$
B.3. $a_3 \mathbf{g} + b_3 \, ad^{3}{}_{\mathbf{f}} \mathbf{g} = [\mathbf{g}, ad^{3}{}_{\mathbf{f}} \mathbf{g}]$

If any of these conditions do not exist, the system of governing equations under investigation is not involutive and, therefore, it is also not input-state linearizable [Slotine e Li, 1991].

6.1 DETERMINE OF THE LIE BRACKETS (GIVEN IN STEP A)

The Lie brackets given in a Step A are:

$$[g, ad_f g] = (\nabla ad_f g)g - (\nabla g)ad_f g = \begin{Bmatrix} A.1_1 \\ A.1_2 \\ A.1_3 \\ A.1_4 \end{Bmatrix} \tag{25}$$

$$[g, ad_f{}^2 g] = (\nabla ad_f{}^2 g)g - (\nabla g)ad_f{}^2 g = \begin{Bmatrix} A.2_1 \\ A.2_2 \\ A.2_3 \\ A.2_4 \end{Bmatrix} \tag{26}$$

$$[ad_f g, ad_f{}^2 g] = (\nabla ad_f{}^2 g)g - (\nabla g)ad_f{}^2 g = \begin{Bmatrix} A.3_1 \\ A.3_2 \\ A.3_3 \\ A.3_4 \end{Bmatrix} \tag{27}$$

where:

$$A.1_1 = A.1_2 = A.1_3 = A.2_1 = A.3_2 = A.3_3 = 0$$

$$A.1_4 = -\frac{c_1 \sin 2x_3}{c_2(c_1 \cos^2 x_3 + I)^2}$$

$$A.2_2 = \frac{2c_1{}^2 x_2 \sin^2 2x_3}{c_2(c_1 \cos^2 x_3 + I)^3}$$

$$A.2_3 = \frac{c_1 \sin 2x_3}{c_2(c_1 \cos^2 x_3 + I)^2}$$

$$A.2_4 = -\frac{c_1((2I^2 + 3Ic_1 + c_1{}^2)x_4 \cos 2x_3 - c_1 c_2 x_4(\cos 4x_3 - 3) + 4CI \cos x_3 \sin x_3 + 4Cc_1 \cos^3 x_3 \sin x_3)}{c_2{}^2(c_1 \cos^2 x_3 + I)^3}$$

$$A.3_1 = \frac{2c_1{}^2 x_2 \sin^2 2x_3}{c_2(c_1 \cos^2 x_3 + I)^3}$$

$$A.3_4 = \frac{c_1{}^2 x_2(2c_1 \sin 2x_3 + 2c_1 \sin^3 2x_3 + c_2 \sin 4x_3)}{c_2{}^2(c_1 \cos^2 x_3 + I)^3}$$

6.2 VERIFYING THE EXISTENCE OF A_i AND B_i (GIVEN IN STEP B)

The idea now, in order to prove the involutivity of the distribution (23), is to prove that the coefficients a_i and b_i in B.1 to B.3 do exist.

The Equation B.1 can be written as:

$$B1 = \begin{bmatrix} 0 & C_{12} \\ C_{21} & 0 \\ 0 & 0 \\ 0 & C_{42} \end{bmatrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ A.1_4 \end{Bmatrix} \tag{28}$$

Solving the system of equations with two unknows given by (28) using the least squares method one obtains:

$$\begin{cases} a_1 = 0 \\ b_1 = 0 \end{cases} \tag{29}$$

which implying the following contradiction:

$$A.1_4 = 0 \tag{30}$$

The Equation B.2 can be written as:

$$B2 = \begin{bmatrix} 0 & C_{13} \\ C_{21} & C_{23} \\ 0 & C_{33} \\ 0 & C_{43} \end{bmatrix} \begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ A.2_2 \\ A.2_3 \\ A.2_4 \end{Bmatrix} \tag{31}$$

Solving the system of equations with two unknows given by (31) using the least squares method one obtains:

$$\begin{cases} a_2 = \frac{A.2_2}{C_{21}} \\ b_2 = 0 \end{cases} \quad (32)$$

wich implying the following contradiction:

$$\begin{cases} A.2_2 = 0 \\ A.2_3 = 0 \end{cases} \quad (33)$$

The Equation B.3 can be written as:

$$B3 = \begin{bmatrix} C_{12} & C_{13} \\ 0 & C_{23} \\ 0 & C_{33} \\ C_{42} & C_{43} \end{bmatrix} \begin{Bmatrix} a_3 \\ b_3 \end{Bmatrix} = \begin{Bmatrix} A.3_1 \\ 0 \\ 0 \\ A.4_4 \end{Bmatrix} \quad (34)$$

Solving the system of equations with two unknows given by (34) using the least squares method one obtains:

$$\begin{cases} a_2 = \frac{A.3_1}{C_{12}} \\ b_2 = 0 \end{cases} \quad (35)$$

and

$$\begin{cases} a_2 = \frac{A.3_4}{C_{42}} \\ b_2 = 0 \end{cases} \quad (36)$$

which implying the following contradiction:

$$\frac{A.3_1}{C_{12}} = \frac{A.3_4}{C_{42}} \quad (37)$$

According with results obtained in B1, B2 and B3, the involutability conditions is not verified and, therefore, the system is nonholonomic. According of the Frobenius’s theorem, if the system is not involutive, this implied that the system is not integrable.

7. CONCLUSION

It is well known that it is not possible to apply any nonlinear control technique in any nonlinear dynamic system.

The possibility of applying a specific nonlinear control technique named feedback linearization in a specific dynamic system is investigated in this work. This dynamic system studied is a first approximation to a helicopter rotor in a static flight.

Feedback linearization is a technique to transform original system models into equivalent models of a simpler form. The central idea is to transform nonlinear systems dynamics into fully or partly linear ones.

The fact that the Lie brackets are linearly independent implies that one can find a basis where the nonlinear system can be controlled by a linear control, because, in this space, the control acts in all of the state variables. Feedback linearization uses mathematical tools from differential geometry, as the concept of Lie derivatives.

According to the results presented here, the controllability conditions to apply this technique are completely satisfied for the system investigated, but the involutive conditions are not satisfied. Thus, according of the Frobenius’s theorem, the proposed system is nonholonomic and not integrable and, therefore, the Feedback Linearization is not applicable.

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