INFLUENCE OF THE SPATIAL DISCRETIZATION IN MODELING AND CONTROL OF THE FLEXIBLE STRUCTURE IN ROTATION CONSIDERING THE FLUID INTERACTION AND HOVERED FLIGHT

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Abstract. *We investigate the control of angular velocity and altitude of a system composed of a central hub and a flexible beam crimped to it. The beam rotates around the hub's axis with an angle of attack relative to the plane of rotation. The control of altitude and angular velocity is designed using a nonlinear control law, known as SDRE (State Dependent Riccati Equation). This control law is applied to the central hub as a torque. We use two techniques for the spatial discretization of the system integro partial differential equations of motion. One such technique is the method of assumed modes and the other is the finite difference method. The Runge-Kutta method is used to obtain numerically the time evolution of the system governing equations. The main objective of this work is to study the effect of the spatial discretization technique in the system time evolution and control. The numerical simulations consider different values for the angle of attack and for the angular velocities. The difference between the system time evolution when using finite difference and the assumed modes for spatial discretization increases when the system non-linearity becomes stronger.*

Keywords: *nonlinear dynamics, rotating flexible beam, fluid-structure interaction, finite differences, assumed modes*

1 INTRODUCTION

Dynamic behavior of flexible structures in rotation represent an area of great research interest to the worldwide scientific community, as well as for the industry due to the wide range of application of these structures. The main objectives of these studies involve, in general, vibration control, lighter structural design for various applications related to areas such as civil, mechanical, robotics, aerospace, ocean and many others. We can cite some of the main applications:

- Lightweight robotic manipulators;
- Solar panels or antennas for satellites;
- Helicopter blades or wind turbines.

According to Laks et al. (2009) , wind power investment worldwide is expected to expand three-fold in the next decade, from about $18 billion in 2006 to $60 billion in 2016. In the U.S., where wind currently only provides about 1% of the nation's electricity, wind has the potential to provide up to 20% of the nation's electricity without major changes to the nation's electricity distribution system. Cao et al. (2007) suggests that rotorcraft with new types of configuration will become the more economical, effective and rapid vehicles for air traffic transportation in the future. Indeed, in an increasingly competitive world, new demands are constantly demanding more security, flexibility, autonomy and efficiency that are closely related on aspects such as stability, dynamics and control techniques.

Çimen (2008) reinforces the advantages of using the method SDRE (State-Dependent Riccati Equation), because it is easily implemented with a simple computational algorithms as Euler and Runge-Kutta method. According to Menon et al. (2004) the computational simplicity of the technique in conjunction with the current technological advances makes it ideal because it can operate in real time. As shown by Langson et al. (2002) at the non-linear problem reference, by Erdem et al. (2004) at the magnetic levitation experiments, by Merttopcuoglu et al. (2007) in control drive a missile guided, by Bogdanov (2007) et al. in testing control small autonomous helicopters, and by Çimen (2009) at the control design of large tankers.

Fenili et al. (2013) discuss that low inherent damping, small natural frequencies, and extreme light weights are among the common characteristics of these systems which make them vulnerable to any external or internal disturbances (such as slewing maneuvers with great velocities, impacts, fluid interaction, etc.). Robotic manipulators with such characteristics are easy to control, need smaller actuators and can reach objectives in a greater workspace since they are thinner and longer than the rigid ones usually used for the same task. In dealing with these kind of rotating structures, the interaction between the angular displacement, θ , also called slewing angle, and the flexible structure deflection variable, $v(x, t)$, can be very important in some cases, as in high angular speed maneuvers. The inclusion of the drag and lift effects, in this case, incorporates, although in a simple manner, the interaction between the structure and the surrounding fluid as, for example, the air or the water. This interaction plays an important role as the fluid dissipates the motion kinetic energy and executes work on the system that can significantly alter the control performance and efficiency.

2 MATHEMATIC MODEL

2.1 The Geometric Model

The geometric model of the dynamic system investigated in this work is presented in figure 1.

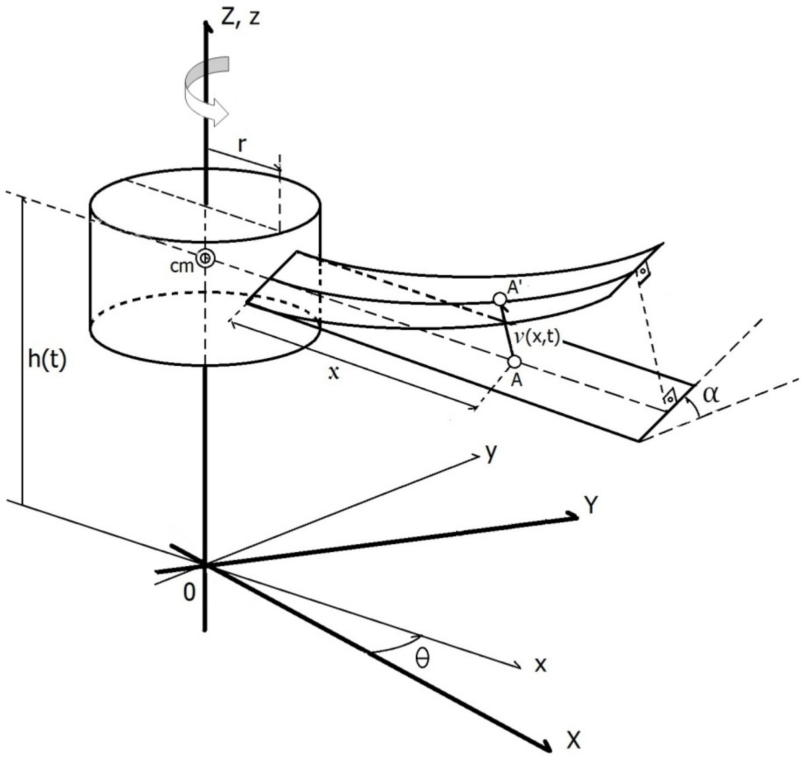

Figura 1: Flexible beam clamped in hub.

This system comprises a rigid hub and a flexible beam-like structure in rotation about Z axis and interacting with a fluid. In this system, the fluid effect on the motion is represented by the drag force, F_{drag}(x, t) and lift L(x,t). These effects are not represented in the figure. The drag and lift forces, as considered in this work, are functions of the velocities ν(x, t) and θ(t). The lift is also a function of the velocity h(t). The model for the lift uses strip theory, and no tridimensional effects are included. In this figure, the inertial axis is represented by XYZ, the moving axis (attached to the rotating axis and moving with it) is represented by xyz.

2.2 Equations of motion

The government equations can be obtained through the Extended Hamilton principle. For this, we need to write the kinetic and potential energy of each system element, and the virtual work of external forces exerted by the fluid. According to Palhares (2015) , the equations of motion from this problem are

$$
\begin{cases}
(m_H + \rho A l)\, \ddot{h} + (m_H + \rho A l) g + \rho A \displaystyle\int_0^l \ddot{v} \cos\alpha dx = \\
\dfrac{1}{2}\rho_f c \displaystyle\int_0^L \left\{ \left([(r+x)\dot{\theta} - \dot{v}\sin\alpha]^2 + [\dot{v}\cos\alpha + \dot{h}]^2 \right) C_L(\alpha) \right\} dx & (1) \\
\\
\left(I_H + \displaystyle\int_0^l \left[(r+x)^2 + v^2 \sin^2\alpha \right] dx \right) \ddot{\theta} + \rho A \displaystyle\int_0^l \left[-\ddot{v}(r+x)\sin\alpha + 2\dot{\theta}\dot{v}v\sin^2\alpha \right] dx = \\
T - \dfrac{1}{2}\rho_f c \sin\alpha \displaystyle\int_0^L \left\{ \left([(r+x)\dot{\theta} - \dot{v}\sin\alpha]^2 + [\dot{v}\cos\alpha + \dot{h}]^2 \right) (r+x) C_D \right\} dx & (2) \\
\\
-\ddot{\theta}(r+x)\sin\alpha + \ddot{h}\cos\alpha + \ddot{v} - \dot{\theta}^2 v \sin^2\alpha + g\cos\alpha + \dfrac{d^2}{dx^2}\left(\dfrac{EI}{\rho A} v'' \right) = \\
\left([(r+x)\dot{\theta} - \dot{v}\sin\alpha]^2 + [\dot{v}\cos\alpha + \dot{h}]^2 \right) \left(\rho_f c C_L(\alpha)\cos\alpha + \rho_f c C_D \sin^2\alpha \right) & (3)
\end{cases}
$$

where h represents the displacement of the hub axis, θ represents the angular displacement of the hub, r represents the radius of the hub, α represents the angle of attack of the beam, ν(x, t) represents the transversal displacement of the beam, ρ represents the density of the material that composes the beam, A represents the beam cross section area, l represents the non deflected length of the beam, E represents the Young's modulus of the material that composes the beam, I represents the moment of inertia of the cross-section area of the beam, ρ_f representing the density of the fluid, c is the beam cross sectional width (mean chord), C_L is the lift coefficient for the beam cross section and C_D representing the drag coefficient.

The boundary conditions for the beam are given by

$$
\begin{cases}
v(0,t) = 0 & (4) \\
v'(0,t) = 0 & (5) \\
v''(L,t) = 0 & (6) \\
v'''(L,t) = 0 & (7)
\end{cases}
$$

3 BEAM DISCRETIZATION

We use two spatial discretization techniques, the finite difference method and the assumed modes method.

For the finite difference discretization, the integrals are aproximated by the numerical quadrature rule known as the Extended-Simpson rule. To advance the system response in time in both dicretizations techniques, we use the 4th-order Runge-Kutta method.

3.1 The Finite Difference Method

One way to perform spacial discretization of the problem was to use the finite differences method to transform the partial differential equation in 2N ordinary differential equations.

$$\frac{d^n f}{dx^n}(x_j) \approx D_k^n f_j = \sum_{l=m}^{m+k-1} \frac{A_l}{(\Delta x)^n} f(x_j + (l-j)\Delta x), \tag{8}$$

A centered operator of fourth order using 5 points that has error of order $O(h^2)$ was used. Applying the finite difference method to in equations (1), (2) and (3), we can represent the system by

$$[M]\begin{Bmatrix} \ddot{h} \\ \ddot{\theta} \\ \ddot{v}_2 \\ \cdots \\ \ddot{v}_{N-1} \\ \ddot{v}_N \end{Bmatrix} + [K]\begin{Bmatrix} h \\ \theta \\ v_2 \\ \cdots \\ v_{N-1} \\ v_N \end{Bmatrix} + \begin{Bmatrix} F_h \\ F_\theta \\ F_2 \\ \cdots \\ F_{N-1} \\ F_N \end{Bmatrix} = 0 \tag{9}$$

where $[M]$ is a mass matrix

$$\begin{cases} M_{1,1} = m_H + m_v & (10) \\ M_{1,j+1} = \rho_j A_j w_j \Delta x \cos\alpha;\ j = 2, \dots, N & (11) \\ M_{2,2} = I_H + \sum_{j=1}^{N} \rho_j A_j (r + (j-1)\Delta x)^2 w_j \Delta x + \sum_{j=1}^{N} \rho_j A_j v_j^2 \sin^2\alpha w_j \Delta x & (12) \\ M_{2,j+1} = -\rho_j A_j (r + (j-1)\Delta x) \sin\alpha w_j \Delta x;\ j = 2, \dots, N & (13) \\ M_{j+1,1} = \rho_j A_j \cos\alpha w_j \Delta x;\ j = 2, \dots, N & (14) \\ M_{j+1,2} = -\rho_j A_j (r + (j-1)\Delta x) \sin\alpha w_j \Delta x;\ j = 2, \dots, N & (15) \\ M_{j+1,j+1} = \rho_j A_j w_j \Delta x;\ j = 2, \dots, N. & (16) \end{cases}$$

$[K]$ is a stiffness matrix

$$K_{i+1,j+1} = \sum_{j=1}^{N} \frac{\partial}{\partial v_n} (D_k^4 v_m)(EI)_j w_j \Delta x;\ m, n = 2, \dots, N, \tag{17}$$

and $\{F\}$ is a force vector

$$
\begin{cases}
F_h = m_H g + m_v g & (18)\\
\quad -\frac{1}{2}\rho_f c \sum_{j=1}^{N} \left\{ \left[\dot{\theta}(r+(j-1)\Delta x) - \dot{v}_j \sin\alpha\right]^2 + \left[\dot{v}_j \cos\alpha + \dot{h}\right]^2 \right\} C_L(\alpha) w_j \Delta x & (19)\\
F_\theta = \sum_{j=1}^{N} \{2\rho_j A_j w_j v_j \dot{v}_j\} \sin^2\alpha \dot{\theta} \Delta x & \\
\quad + \frac{1}{2}\rho_f c \sum_{j=1}^{N} \left\{ \left[\dot{\theta}(r+(j-1)\Delta x) - \dot{v}_j \sin\alpha\right]^2 + \left[\dot{v}_j \cos\alpha + \dot{h}\right]^2 \right\} & \\
\quad (r+(j-1)\Delta x) \sin\alpha C_D w_j \Delta x - T & (20)\\
F_l = -\rho_l A_l w_l \Delta x \left[v_l \sin^2\alpha \dot{\theta}^2 - g\cos\alpha\right] & \\
\quad -\frac{1}{2}\rho_f c \left\{ \left[\dot{\theta}(r+(l-1)\Delta x) - \dot{v}_l \sin\alpha\right]^2 + \left[\dot{v}_l \cos\alpha + \dot{h}\right]^2 \right\} & \\
\quad \left(C_L(\alpha)\cos\alpha + C_D \sin^2\alpha\right) w_l \Delta x;\ l = 2, \ldots, N & (21)
\end{cases}
$$

3.2 The Assumed Modes Method

Another way to perform the spatial discretization is the assumed modes method, that is, assuming that the solution is a product of ordinary functions.

$$f(x,t) = \sum_{j=1}^{n} \Phi_j(x) q_j(t) \tag{22}$$

We considered only the the first mode of the clamped-free beam. Thus, all equations has become ordinary differential equations, and the system assumes the form

$$\begin{bmatrix} M_{11} & 0 & M_{13} \\ 0 & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\theta} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_{33} \end{bmatrix} \begin{Bmatrix} h \\ \theta \\ q \end{Bmatrix} + \begin{Bmatrix} F_h \\ F_\theta \\ F_q \end{Bmatrix} = 0 \tag{23}$$

$$
\begin{cases}
M_{1,1} = m_H + m_v & (24)\\
M_{1,3} = \int_0^L \rho_j A_j \Phi \cos\alpha w_j dx & (25)\\
M_{2,2} = I_H + \int_0^L \rho_j A_j (r + (j-1)\Delta x)^2 \Phi^2 w_j \Delta x + \int_0^L \rho_j A_j q_j^2 \Phi^2 \sin^2\alpha w_j dx & (26)\\
M_{2,3} = -\int_0^L \rho_j A_j (r + (j-1)\Delta x)^2 \Phi^2 \sin\alpha w_j dx & (27)\\
M_{3,1} = \int_0^L \rho_j A_j \Phi \cos\alpha w_j dx & (28)\\
M_{3,2} = -\int_0^L \rho_j A_j (r + (j-1)\Delta x)^2 \Phi^2 \sin\alpha w_j dx & (29)\\
M_{3,3} = \int_0^L \rho_j A_j \Phi^2 w_j dx & (30)
\end{cases}
$$

$$
K_{3,3} = \int_0^L \rho_j a_1^4 A_j \Phi \tag{31}
$$

$$
\begin{cases}
F_h = m_H g + m_v g \\
\quad - \frac{1}{2}\rho_f c \int_0^L \Big\{ \dot\theta^2 (r + (j-1)\Delta x)^2 \Phi^2 - 2\dot\theta \dot q \sin\alpha (r + (j-1)\Delta x)\Phi^2 + \dot q_j^2 \Phi^2 \\
\quad + 2\dot q \dot h \cos\alpha \Phi + \dot h^2 \Big\} C_L(\alpha) w_j dx & (32)\\
F_\theta = \int_0^L \{2\rho_j A_j w_j q_j \dot q_j\} \sin^2\alpha \dot\theta \Delta x \\
\quad + \frac{1}{2}\rho_f c \int_1^L \Big\{ \dot\theta^2 (r + (j-1)\Delta x)^3 \Phi^3 - 2\dot\theta \dot q \sin\alpha (r + (j-1)\Delta x)^2 \Phi^3 + \dot q_j^2 (r + (j-1)\Phi^3 \\
\quad + 2\dot q \dot h \cos\alpha (r + (j-1)\Delta x)\Phi^2 + \dot h^2 (r + (j-1)\Delta x)\Phi \Big\} \sin\alpha C_D(\alpha) w_j dx - T & (33)\\
F_l = -\int_0^L \rho_j A_j \left[q \sin^2\alpha \dot\theta^2 \Phi^2 - g\cos\alpha \Phi \right] w_j \Delta x \\
\quad \frac{1}{2}\rho_f c \int_0^L \Big\{ \dot\theta^2 (r + (j-1)\Delta x)^2 \Phi^3 - 2\dot\theta \dot q \sin\alpha (r + (j-1)\Delta x)\Phi^3 + \dot q_j^2 \Phi^3 + \\
\quad 2\dot q \dot h \cos\alpha \Phi^2 + \dot h^2 \Phi \Big\} \left(C_L(\alpha)\cos\alpha + C_D(\alpha)\sin^2\alpha \right) w_j dx & (34)
\end{cases}
$$

3.3 The State Space Representation

To apply the Runge-Kutta method it is necessary to reduce the 2nd order ODE's to 1st order. Thus, we suggest the following transformation variables:

$$\left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ \vdots \\ x_{2j+1} \\ x_{2j+2} \end{array} \right\} = \left\{ \begin{array}{c} h \\ \dot{h} \\ \theta \\ \dot{\theta} \\ v_2 \\ \dot{v}_2 \\ \vdots \\ v_j \\ \dot{v}_j \end{array} \right\} \tag{35}$$

Since the system equations are coupled, it was convenient to perform some operation to facilitate the numerical treatment of the solution. The set of operations presented below transforms the equations into a state-space representation

$$\left\{ \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{2j-1} \\ \dot{x}_{2j} \end{array} \right\} = - \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & M_{11} & & 0 & M_{1j} \\ & \vdots & \ddots & \vdots & \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & M_{j1} & & 0 & M_{jj} \end{bmatrix}^{-1} \begin{bmatrix} 0 & -1 & \cdots & 0 & 0 \\ K_{11} - \frac{F_1}{x_1} & 0 & & K_{1j} & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & 0 & -1 \\ K_{j1} - \frac{F_1}{x_1} & 0 & & K_{jj} & 0 \end{bmatrix} \left\{ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_{2j-1} \\ x_{2j} \end{array} \right\}$$

4 THE STATE-DEPENDENT RICCATI EQUATION (SDRE) CONTROL

The state-dependent Riccati equation (SDRE) approach to nonlinear system control relies on representing a nonlinear system's dynamics with state-dependent coefficient matrices that can be inserted into state-dependent Riccati equations to generate a feedback law. The main idea of this method is to represent the nonlinear system:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \tag{36}$$

According to equation (36), for the dynamic system studied in this work, the matrix B is not state dependent.

The feedback law is given by

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{37}$$

where P(x) is obtained from the SDRE

$$P(x)A(x) + A^T(x)P(x) + Q(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) = 0 \tag{38}$$

5 NUMERICAL SIMULATION

The values adopted for the parameters and used in the numerical simulations they consider a steel beam and the fluid is air. This values are presented in Table (1)

Tabela 1: Structure and Fluid constants

Parameter	Value	Unit
I	$1,2358.10^{-10}$	m^4
E	$2,1.10^{11}$	Pa
g	$9,81$	m/s^2
ρ	7860	Kg/m^3
A	$9,7500.10^{-5}$	m^2
L	1	m
r	0.1	m
c	0.025	m
ρ_f	1.184	Kg/m^3
C_L	$2\pi\alpha$	-
C_D	1.28$\sin\alpha$	-

and the initial conditions are presented in Table (2)

Tabela 2: Initial Conditions

Parameter	Value	Unit
h	0	m
$\dot{h}$	0	m/s
θ	0	º
$\dot{\theta}$	0	rad/s
$v(x)$	$-0.0630\cos\alpha$	m
$\dot{v}(x)$	0	m/s

This initial condition for $v(x)$ represents the equilibrium condition of beam deflexion. For the B controller matrix, R and Q gain matrix are considered

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & & 1 & 0 \\ 0 & M_{11} & 0 & M_{12} & 0 & M_{13} & & 0 & M_{1j} \\ 1 & 0 & 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & M_{21} & 0 & M_{22} & 0 & M_{23} & & 0 & M_{2j} \\ 1 & 0 & 1 & 0 & 1 & 0 & & 1 & 0 \\ 0 & M_{31} & 0 & M_{32} & 0 & M_{33} & & 0 & M_{3j} \\ & & & \vdots & & & \ddots & & \vdots \\ 1 & 0 & 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & M_{j1} & 0 & M_{j2} & 0 & M_{j3} & & 0 & M_{jj} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ & & & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \end{bmatrix} \tag{39}$$

$$R = 200 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 \\ & & & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 1 \end{bmatrix} \tag{40}$$

$$Q = 1000 \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 25 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 25000 & & 0 & 0 \\ & & & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 0 & 25000 \end{bmatrix} \tag{41}$$

The objetive is to control the $\dot{\theta}$ variable because maintaining constant rotation, the lift force will be constant and the set will hover at a certain altitude.

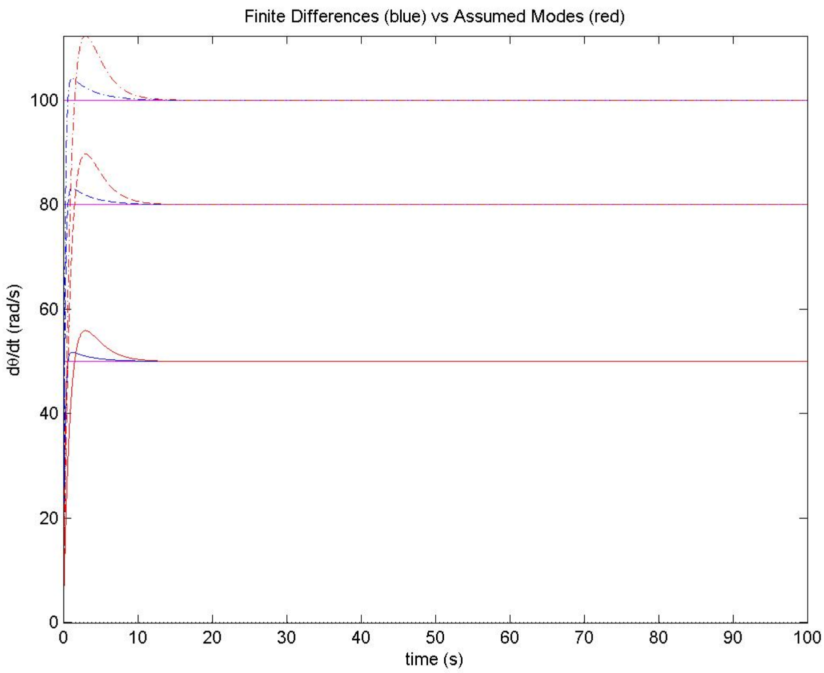

Figura 2: Angular velocity $\dot{\theta}$ for $\alpha = 1^{o}$ - 50rad/s(continuous) vs 80rad/s(dashed) vs 100rad/s(dash-dot)

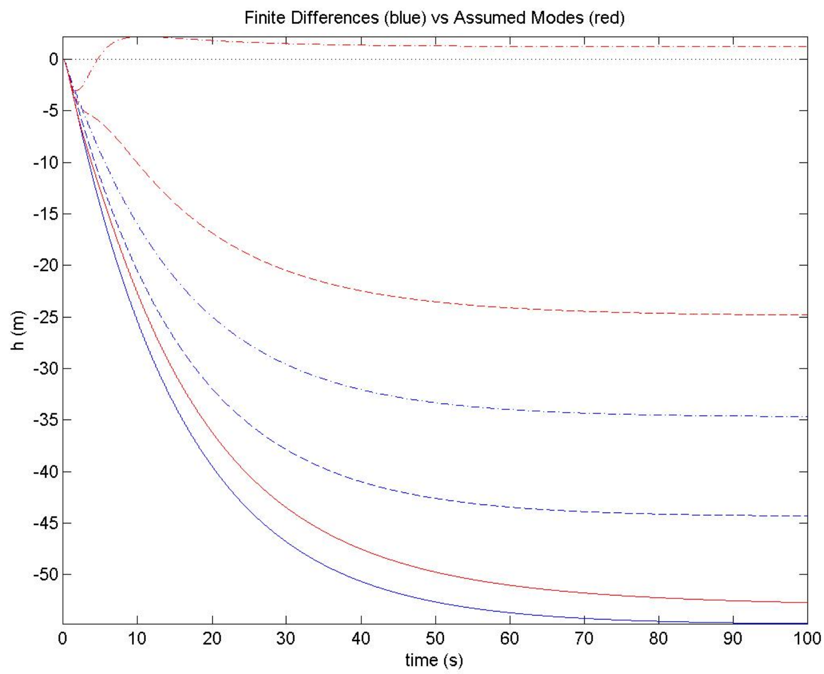

Figura 3: Height h for $\alpha = 1^{o}$ - 50rad/s(continuous) vs 80rad/s(dashed) vs 100rad/s(dash-dot)

The beam used in this model have rectangular thin section in a manner that the aerodynamic coefficient was approximated by a flat plate. Therefore, the theory used does not allow $\alpha > 5^o$.

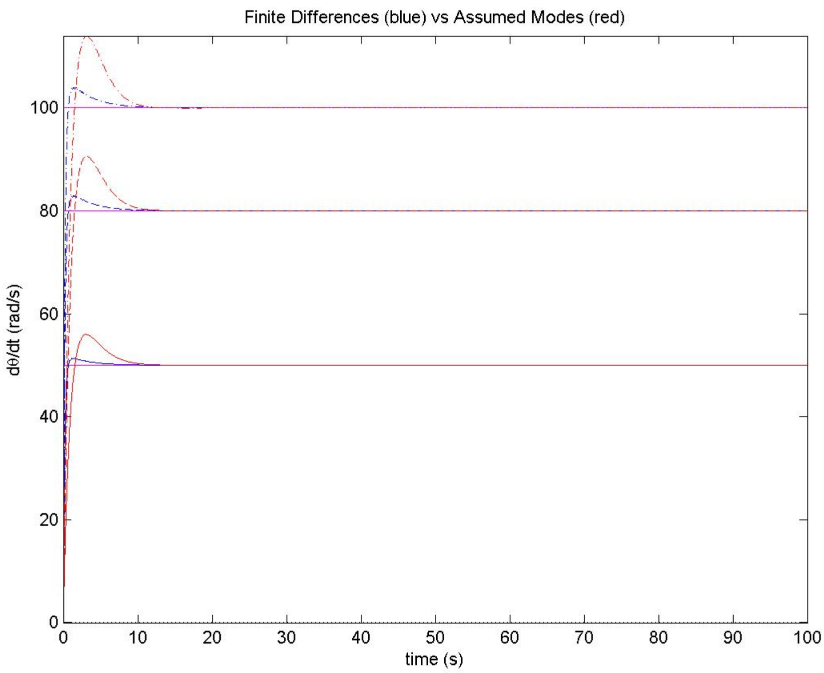

Figura 4: Angular velocity $\dot{\theta}$ for $\alpha = 5^o$ - 50rad/s(continuous) vs 80rad/s(dashed) vs 100rad/s(dash-dot)

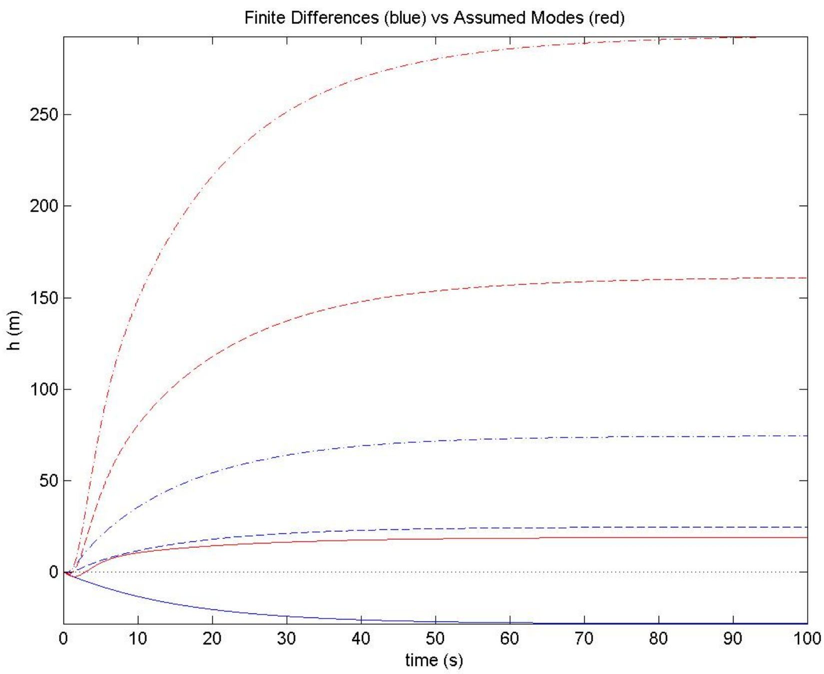

Figura 5: Height h for $\alpha = 5^o$ - 50rad/s(continuous) vs 80rad/s(dashed) vs 100rad/s(dash-dot)

The solution by finite differences gives us the evolution of the all nodes of the beam,

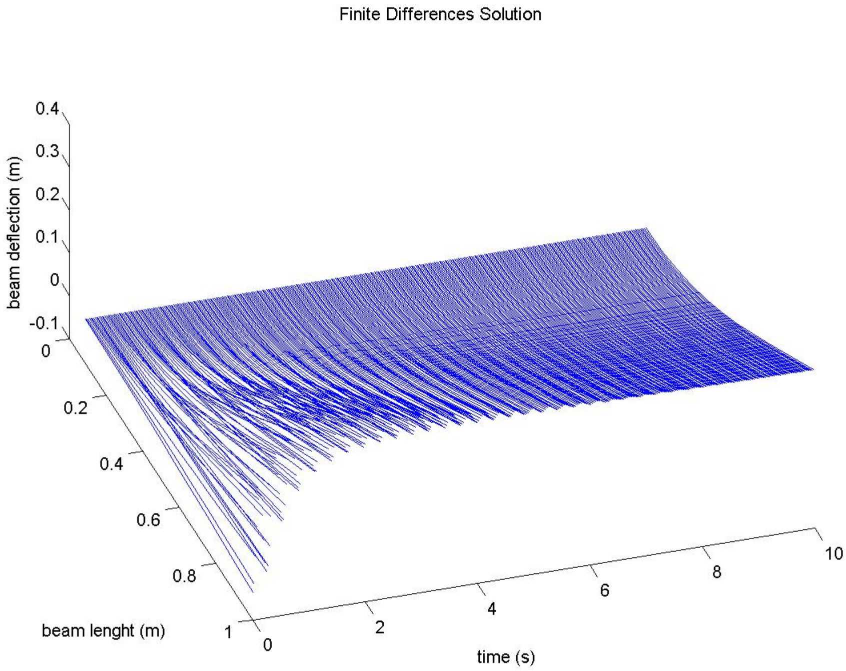

Figura 6: Temporal evolution of the beam deflexion

but, for compare with solution by assumed modes, is necessary consider only the last node v_N

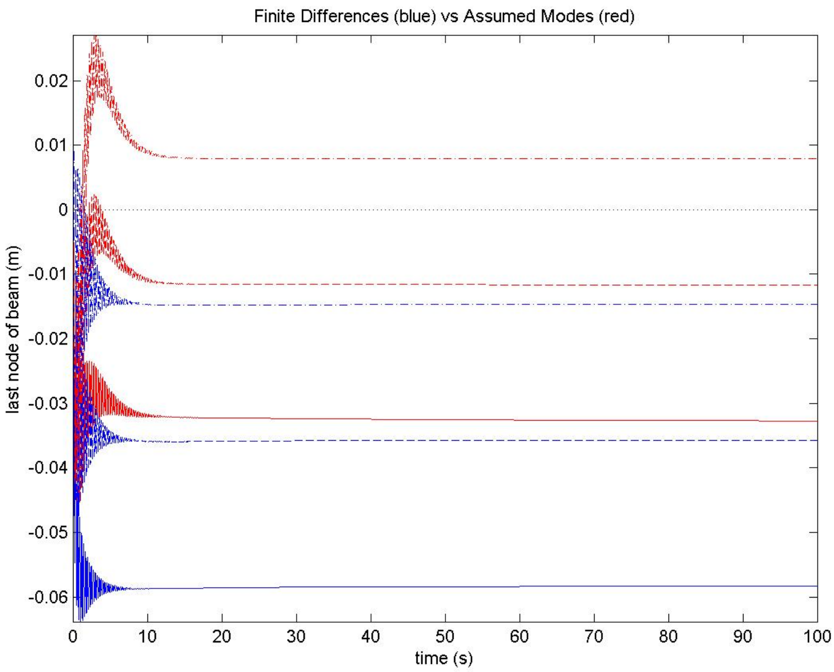

Figura 7: Node v_N deflexion for $\alpha = 1^o$ - 50rad/s(continuous) vs 80rad/s(dashed) vs 100rad/s(dash-dot)

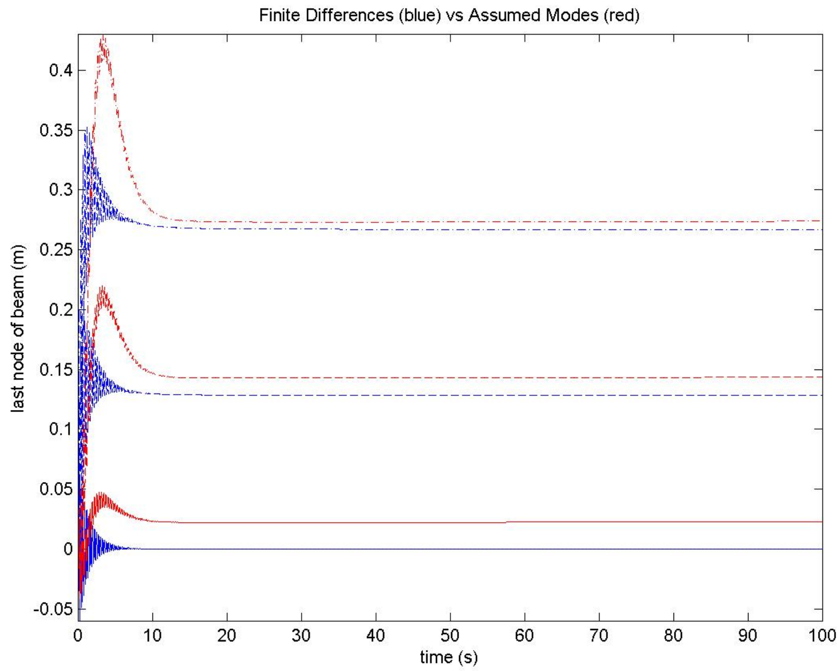

Figura 8: Node v_N deflexion for $\alpha = 5^{\text{o}}$ - 50rad/s(continuous) vs 80rad/s(dashed) vs 100rad/s(dash-dot)

It is important to realize that $\alpha = 5^{\text{o}}$ and $\dot{\theta} = 100\text{rad/s}$, the deflection is greater than 15% of the beam length, so the beam theory linear curve is not a good approximation.

6 CONCLUSION

In this paper we propose a dynamic model of a flexible beam clamped in a hub and under the influence of aerodynamic forces of drag and lift. This system of equations, while possessing simplifying assumptions, consider the fundamental physical processes of a dynamic flexible beam in rotation, considering their interaction with the fluid in which it is immersed, where the drag and lift forces are modeled using strip theory. The purpose is to develop a model of fluid-structure interaction that contains the basic physical characteristics of the problem, but at the same time, can be solved using simple numerical techniques

- The responses obtained from the assumed modes method are overestimated in relation to finite difference method.
- The error between the two methods grows as we increase the angle of attack and speed, due to increased nonlinearities.
- For each angle of attack value, exist a minimum speed for the lift force to balance the weight.
- For cases in which the beam deflection is greater than 20% it is necessary to use nonlinear beam theory.

The simulations show the robustness of the control SDRE technique, since even when subjected to very significant non-linearity, the system was controlled very efficiently. Although the assumed modes method carry many errors in relation to finite differences method, the control was able to make a stable dynamic even when the errors involved in the approach of the lift and drag coefficients were extreme.

Future work will be a system stability analysis, seeking to find optimal gains as well as working in the transient part of the response, using the Ricatti differential equation. Furthermore, an aerodynamic profile will be proposed in order to replace the flat plate profile, to make the effects of flow and fluid-structure interaction of a real model and consider the three-dimensional effects such as vortex sheet and its memory effect on the flow. From the estimation of system parameters (mass, length, structure coefficients: damping, stiffness, lift and drag, etc...), is expected to find a faithful representation of the dynamics of a rotor and then apply the proposed control technique in a real prototype.

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